LEVEL

AD A082169

STATISTICAL IMAGE MODELING

DTIC
ELECTED
MAR 24 1980
S
C

by

C. H. /Chen

Department of Electrical Engineering
Southeastern Massachusetts University
North Dartmouth, Massachusetts 02747

ABSTRACT

Image models are very useful for image coding, compression, segmenta-
tion, interpretation as well as image enchancement and restoration. For
many images in practical applications, statistical information is most
important. This report deals with the fundamental statistical theory of
image models including the topics of contextual analysis, stochastic random
field, the local and global properties of the random field, ARMA systems,
and the applications of the statistical image models.

APPROVED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED.

DDC FILE COPY.

79 11 07

80 3 20 059

STATISTICAL IMAGE MODELING

I. INTRODUCTION

The way a digital image is processed depends largely on how it is
modeled. ~~Generally speaking~~ image models are useful in image coding,
compression, interpretation, classification, texture characterization as
well as image enchancement and restoration. Both statistical and struc-
tural models of images have been considered as the images contain both
statistical and structural information. Purely structural models are
too regular to be interesting. In most practical applications especially
in the defense area, the statistical information is most important. With
these applications in mind, this report deals with the fundamental theore-
tical topics in statistical image modeling. An extensive list of references
is provided to cover many publications in this important area.

II. THE CONTEXTURAL ANALYSIS

Consider a digital image with M x N picture elements, i.e., M rows
and N columns. A simple approximation of contextual dependence for the
two-dimensional patterns is called Markov mesh [1], which is considered
as a two-dimensional Markov chain. Assume that the image is partitioned
into m x n subimages. Then this two-dimensional Markov chain is charac-
terized by a transition probability matrix $P_{m,n}$ defined as

$$P_{m,n} = \begin{pmatrix} P_{1,1} & P_{1,2} & --- & P_{1,b-1} & P_{1,b} & --- & P_{1,n} \\ P_{2,1} & P_{2,2} & --- & P_{2,b-1} & P_{2,b} & --- & P_{2,n} \\ \cdot & \cdot & --- & \cdot & \cdot & --- & \cdot \\ \cdot & \cdot & --- & \cdot & \cdot & --- & \cdot \\ \cdot & \cdot & --- & \cdot & \cdot & --- & \cdot \\ P_{a-1,1} & P_{a-1,2} & --- & P_{a-1,b-1} & P_{a-1,b} & --- & P_{a-1,n} \\ \cdot & \cdot & --- & \cdot & \cdot & --- & \cdot \\ \cdot & \cdot & --- & \cdot & \cdot & --- & \cdot \\ \cdot & \cdot & --- & \cdot & \cdot & --- & \cdot \\ P_{m,1} & P_{m,2} & --- & P_{m,b-1} & P_{m,b} & --- & P_{m,n} \end{pmatrix} \tag{1}$$

where $P_{ij} = P(x_j|x_i)$ is the transition probability. Here x_i, x_j are the vector measurements of subimages. It can be shown that for binary random patterns,

$$P(P_{a,b}|P_{m,n}) = P\left[P_{a,b} \left| \begin{array}{ccc} P_{a-1,b-1} & P_{a-1,b} & P_{a-1,b+1} \\ P_{a,b-1} & & P_{a,b+1} \\ P_{a+1,b-1} & P_{a+1,b} & P_{a+1,b+1} \end{array} \right. \right] \qquad (2)$$

That the transition probability $P_{a,b}$ depends only on the transition probabilities of the neighboring submiages is a very important result in contextual analysis. The above result may be generalized to grayscale pictures. In general, the dependence on the eight neighboring subimages is most important in contextual analysis.

The use of neighbor dependence approximation for the spatial patterns was first studied by Chow [2]. The tree dependence he considered [3] can also be generalized to spatial patterns. In this case each subimage will depend on certain surrounding subimages in addition to the eight neighboring subimages. For image interpretation or classification, the compound decision theory provides a theoretical framework for decision making using the contextual information. However, the practical implementation of the compound decision rule has been limited to Markov dependence or neighbor dependence. Assume that a subimage depends only on its four adjacent subimages in the east-west and north-south directions, then the decision rule is to choose the class that maximizes the likelihood function [4].

$$P(\theta)P(x|\theta) \prod_{i=1}^{4} P(n_i|\theta) \qquad (3)$$

where x and x_i correspond to vector measurements of the subimage under consideration and its neighboring subimage respectively and $\theta = 1, 2, --, n$ with m being the number of classes. Here

$$P(x_i \mid \theta) = \sum_{j=1}^{m} P(n_i \; \theta_j) P(\theta_j \mid \theta) \tag{4}$$

where $\theta_j = 1, 2, \ldots m$ and $P(\theta_j \mid \theta)$ is the transition probability which is usually determined experimentally. If the four subimages in the four corners are also considered, then the likelihood function for the eight-neighbor dependence case is

$$P(\theta) \; P(x \mid \theta) \; \prod_{i=1}^{8} \; p(x_i \mid \theta) \tag{5}$$

where $P(x_j \mid \theta)$ is also given by Eq. (4). The simple result given by Eqs. (3) and (5) is due to the assumption of conditional independence between the subimage and the neighboring subimages and the assumption that the contribution due to the adjacent subimages is independent of that due to subimages in four corners. Both assumptions are reasonable in theory though quite restrictive in practice. For example, the occurrence of one class at one subimage will affect the occurrences of its neighboring classes. Obviously, the results are not valid for dependence on arbitrary set of subimages in the neighborhood. Another problem with the compound decision rule is that the conditional probability densities are usually not available and the a priori information may not be accurate. Experimental results based on Gaussian assumption of probability densities have consistently demonstrated the performance improvement with the use of contextual information [4], [5]. Empirically it is possible to determine these densities from the histogram of gray levels of each subimage, which corresponds to an unconditional probability density [6].

As the statistical contextual analysis based on the neighborhood dependence model described above is quite restrictive in practice, further development in image modeling is much needed for contextual analysis.

- 3 -

III. STOCHASTIC RANDOM FIELD

The Markov random field is the most typical assumption in statistical models. Wong [7] considered the properties of a two-dimensional random field having finite first and second moments. He found that there is no continuous Gaussian random field of two dimensions which is both homogeneous and Markov of degree 1. A homogeneous random field has a covariance function that is invariant under translation as well as rotation. Woods [8] considered a more general definition of Markov mesh than that discussed in the last section. Hassner and Sklansky [9] also discussed a Markov random field model for images. They presented an algorithm that generates a texture from an initial random configuration and a set of independent parameters that specify a consistent collection of nearest neighbor conditional probabilities which characterize the Markov random field.

For many practical applications such as in military area, the homogeneous random field assumption is not valid because of the object boundaries. Nahi and Jahanshashi [10] suggested modelling the image as a background statistical process combined with a set of foreground statistical processes, each replacing the background in the regions occupied by the objects being considered. Let

$b_{(m,n)}$ = gray level at the mth row and nth column,

$\gamma_{(m,n)}$ = a binary function carrying the boundary information,

b_b = a sample gray level from the background process,

b_o = a sample gray level from the object process, and

v = a sample gray level from the noise process,

then the model can be written as

$$b_{(m,n)} = \gamma_{(m,n)} b_{o(m,n)} + [1 - \gamma_{(m,n)}] b_{b(m,n)} + v_{(m,n)} \tag{6}$$

where γ incorporates the assumption of first order Markov process on the object boundaries.

Huang modeled image scan lines as a Markov jump process [11]. This model led to non-linear noise reduction and image segmentation algorithms that are superior to linear techniques. The recursive calculation of a conditional probability involving the boundary component of the scan lines was the key to the non-linear algorithms. Modestino [12] modeled the image as a marked point process evolving according to a spatial parameter. In another approach the image is considered as a spatially variant linear system superimposed by non-linear elements corresponding to object boundaries. Ingle and Woods [13] considered the use of a bank of Kalman filters corresponding to various correlation directions and demonstrated a considerable improvement in the visual quality compared with linear constant coefficient Kalman filtering. Chen [14] has employed an adaptive Kalman filtering that operates a generalized likelihood ratio test in parallel with the Kalman filter. An object boundary corresponds to a state jump that is detected and used to update the Kalman filter. It appears that both textural and temporal variations can be properly taken into account in the image enhancement.

IV. LOCAL AND GLOBAL MODELS

Local statistical image models emphasize on the use of local statistics while global models attempt a description of the random field by using the information from the entire field. In the absence of any knowledge or as assumption about the global process underlying a given image, one may attempt to describe the joint probability density of the gray level or other properties of the picture elements. To do this for the entire image involves extremely high dimensional space which is unrealistic. It would be easier to consider a small neighborhood. However, even for a 3 x 3 neighborhood, a nonparametric representation in a 9 dimensional space along with the asso-

ciated sample size and storage problems still presents serious difficulty.

Thus even for local models, it would be desirable to "compress" the local properties to a low dimensional space. Local description of co-occurrence statistics for textures (e.g. [15]) uses only 2 x 1 neighborhoods. Different features can then be derived from the co-occurrence matrix for texture classification.

Most of the local models, however, use conditional probabilities of picture elements within a window, instead of their joint probability distributions described above. The Markov dependence assumption will make a picture element depend upon its neighbors. Let r and s be the row number and the column number associated with a picture element x. A conditional nearest-neighbor model is

$$P \, [x_{rs}/\text{all other values}]$$
$$= \, P \, [x_{rs}/x_{r-1,s}, \, x_{r+1,s}, \, x_{r,s-1}, \, x_{r,s+1}] \tag{7}$$

and is also known as a Markov field. An efficient procedure to take into account the local dependence is the statistical theory of nearest-neighbor systems on a lattice [16]. If we consider the four neighbors in the east-west west and north-south directions, then we have a non-causal model given by
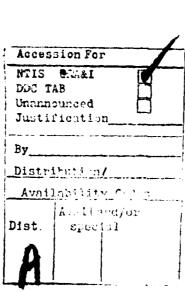
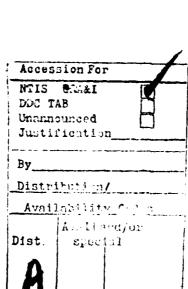$$x_{rs} = \beta_1(x_{r-1,s} + x_{r+1,s}) + \beta_2(x_{r,\,s+1}) + x_{r,s-1} + Y_{rs} \tag{8}$$

where $r = 1, \, 2, \, \ldots, \, M$, $j = 1, \, 2, \, \ldots, \, N$; β_1 and β_2 are the coefficients to be determined; and $\{Y_{rs}\}$ is an uncorrelated Gaussian noise process with $E[Y_{rs}] = 0$, var $[Y_{rs}] = \sigma_i^2$, $i = 1, \, 2, \ldots,$ m where m is the number of pattern classes under consideration. The coefficients β_1 and β_2 may also differ among various classes. A special case of Eq. (8) is the causal model given by

$$x_{rs} - \mu_i = \beta_i \, \{(x_{r-1,s} - \mu_i) + (x_{r,s-1} - \mu_i)\} + Y_{rs} \tag{9}$$

Addendum

Some new image processing result based on the adaptive Kalman filtering [14] is shown on this page. Figure A is the original scene. The object area with two airplanes and with additive noise is shown on Figure B. Figure C is the result of adaptive filtering. All pictures are in binary (two-level) form.

(A)

(B)

(C)

Each class ω_1 corresponds to a set of parameters $(\mu_i, \beta_1, \sigma_1)$. The parameter values are given or estimated for each class. Classification consists in deciding which of the given sets of parameter values best describes, in terms of probabilities, the image to be classified. It is noted that the model given by Eq. (9) permits discrimination between classes ω_i and ω_j even when $(\mu_i, \sigma_i) = (\mu_j, \sigma_j)$ so long as $\beta_i \neq \beta_j$. That is the model performs classification by using the information about the inter-pixel correlation as well as the mean and standard deviation of gray levels. Both Eqs. (8) and (9) represent first order autoregressive model where the autoregression parameters β_i describe the spatial correlation. An alternate expression for the model described by Eq. (9) is

$$x_{rs} = \alpha_i + \beta_i [x_{r-1,s} + x_{r,s-1}] + Y_{rs}$$

$$= \alpha_i + \beta_i Z_{r,s} + Y_{rs}$$

where $\alpha_i = (1-2\beta_i)\mu_i$, and $Z_{r,s} = x_{r-1,s} + x_{r,s-1}$. Here α_i and β_i can be estimated from ordinary simple linear regression by the method of least squares. The maximum likelihood decision rule can be used to classify x_{rs} which follows the Gaussian probability with all parameters represented by their least squares estimates.

It is noted that one difficulty with the model given by Eq. (8) is that, even if the Y_{rs} and hence the x_{rs} are Gaussian, the estimation of β_1 and β_2 from data is not a simple least-squares problem, because the Jacobian of the transformation from the noise variable Y_{rs} to the observation x_{rs} is difficult to evaluate. If the density function of the finite Fourier transform of x_{rs} is considered, then the Jacobian of transformation will be unity [17]. Eqs. (8) and (9) can be generalized to higher order models. For the second order model, the picture elements identified as 1's and 2's in the following figure should be included in the linear model.

```
.   .   2   .   .

.   2   1   2   .

2   1   0   1   2

.   2   1   2   .

.   2   1   2   .

.   .   2   .   .
```

For the nth order model, the picture elements up to a distance of n should be included in the model. The expression is given by

$$x_{rs} = \sum_{i=1}^{n} \beta_i \, (x_{r-1,s} + x_{r+1,s}) + \sum_{i=1}^{n} \beta_i' \, (x_{r,s-i} + x_{r,s+1}) + Y_{rs} \quad (11)$$

where the parameters β_i, β_i' can be estimated by using the maximum likelihood principle [18] if only casual terms are used in Eq. (11).

For the global models considered, the Gaussian model is an oversimplification [19] even though it is mathematically tractable. The stationary Gaussian assumption requires that the mean vector of gray levels be a vector of identical components. Hunt [20] suggested to use a nonstationary Gaussian model which allows the mean vector to have unequal components.

It would be desirable to include some structural property in the in the global model. Matheron [21] used the term "regionalized variables" to emphasize the particular features of the picture elements whose complex mutual correlation reflects the structure of the underlying phenomenon. He assumed weak stationarity of the increments in the gray levels between picture elements. The second moment of the increments in the gray levels between picture elements at an arbitrary distance, called the variogram, is used to reflect the structure of the field. Knowledge of the variogram

is useful for the estimates of many global and local properties of the field. A characterization similar to the variogram is given by the auto-correlation function. However for real imagery good functional forms of variogram and autocorrelation are seldom available. A reasonable approximation must be sought between the functional form cf the image model and the real data considered.

V. THE ARMA SYSTEMS

Eq. (11) is the autoregressive model in its general form. A more general parametric model is called the autoregressive moving average (ARMA) system which replaces Y_{rs} in Eq. (11) by a finite number of previous Y_{rs} values. Of course only the causal terms are taken in Eq. (11). The resulting ARMA or mixed model can provide a good representation of the image with properly chosen coefficients. However an autoregressive model of sufficient order should be adequate.

To determine the order of the model, a maximum likelihood decision rule can be used for choices of neighbors [17]. A simpler procedure is to use the Akaike Information Criterion (AIC) in each of the two dimensions and thus the window size can be determined [22] by the final prediction error. We consider two lines parallel to the horizontal and vertical axes passing a point (i,j), and employ one-dimensional estimators.

$$\hat{x}_{i,j} = \sum_{p=1}^{M} \beta_i^{(M)} x_{i-p,j}$$

$$\hat{x}_{i,j} = \sum_{q=1}^{N} \beta_i^{(N)} x_{i,j-q} \tag{12}$$

where M and N are the window sizes in the horizontal and vertical directions respectively. Let $S_{min}^{(M)}$ and $S_{min}^{(N)}$ be the minimum estimation errors. We deter-

mine the optimum values of M and N so that the two criterion functions

$$FPE\ (M) = \frac{I + M + 1}{I - M - 1}\ S_{min}^{(M)}$$

$$FPE\ (N) = \frac{J + M + 1}{J - M - 1}\ S_{min}^{(N)} \qquad (13)$$

are minimized. Here the original image size is assumed as I x J. A more accurate procedure would consider the two dimensions jointly and may lead to a smaller window size because of the correlation among the adjacent picture elements. Although, the AIC may lead to inconsistent result, it is by far the simplest criterion for determining the window size.

The coefficients in the autoregressive model should be determined from the autocorrelation function. However it would be desirable to develop an efficient two-dimensional Levinson recursion to compute the coefficients. The frequency domain analysis and the maximum entropy method are alternative procedures for estimation of coefficients.

The study of ARMA systems for image analysis is still at its infancy. Further development of ARMA systems is much needed for texture characterization and object boundary extraction.

VI. APPLICATIONS OF STATISTICAL IMAGE MODELS

In the ARMA systems, images are described by a few coefficients or parameters which may be coded for image transmission instead of transmitting the whole picture. This represents an important approach to image compression. Although a limited amount of effort has been made so far in this direction [23] , development in two-dimensional ARMA models will find great applicability in image transmission.

For the image interpretation and classification, image models must emphasize on discrimination information such as the local statistics as the object and background must be statistically different. Different objects must have different model parameters. Although most image models are designed to represent or characterize an image rather than to discriminate among different objects or classes, useful discrimination information is contained in the image models.

Because of a large variety of images, many different image models are available. Statistical models are most effective for images which are rich in texture. The image processing techniques employed are often determined by image modeling. For example, Kalman filtering is particularly suitable for images modeled as a spatially variant linear system with additive noise. For images modeled with a multiplicative disturbance, a different image processing technique is required. On the other hand, preprocessed images usually make image modeling easier. Image models also provide useful knowledge about the image for image segmentation and restoration.

In summary image modeling in general and statistical image modeling in particular provides an abstraction of rich information of various nature in the images and should be considered as an integral part of image analysis and synthesis.

REFERENCES

1. K. Abend, T. J. Hartley, and L. II. Kanal, "Classification of Binary Random Patterns," IEEE Trans. on Information Theory, IT-11, pp. 538-544, October, 1965.

2. C. K. Chow, "A Recognition Method Using Neighbor Dependence," IRE Trans. on Electronic Computers, EC-11, pp. 683-690, October, 1962.

3. C. K. Chow, "A Class of Non-Linear Recognition Procedures," IEEE Trans. on Systems Science and Cybernetics, Vol. 1, December, 1966.

4. J. R. Welch and K. G. Salter, "A Context Algorithm for Pattern Recognition and Image Interpretation," IEEE Trans. on SMC, vol. SMC-1, pp. 24-30, January, 1971.

5. T. S. Yu and K. S. Fu, "Statistical Pattern Recognition Using Contextual Information," TR-EE 78-17, Purdue University, March, 1978.

6. C. II. Chen, "A New Look at the Statistical Pattern Recognition," IEEE Computer Society Workshop on Pattern Recognition and Artificial Intelligence, April, 1978.

7. E. Wong, "Two-Dimensional Random Field and Representations of Images," SIAM Journal of Appl. Math., Vol. 6, pp. 756-770, 1968.

8. J. W. Woods, "Two Dimensional Discrete Markovian Fields," IEEE Trans. on Information Theory, IT-18, pp. 232-240, 1972.

9. M. Hassner and J. Sklansky, "Markov Random Field Models of Digitized Image Texture," Proc. of the 4th International Joint Conference on Pattern Recognition, pp. 538-540, November, 1978.

10. N. E. Nahi and M. H. Jahanshahi, "Image Boundary Estimation," IEEE Trans. on Computers, Vol. C-26, pp. 772-781, August, 1977.

11. T. S. Huang and P. A. Wintz, "Image Analysis and Modeling," AD-A023 633, Final Report, March, 1976.

12. J. W. Modestino and R. W. Fries, "Stochastic Models for Images and Applications" in Pattern Recognition and Signal Processing," NATO ASI E29, Sijthoff & Noordhoff International Publishers, December, 1978.

13. V. K. Ingle and J. W. Woods, "Multiple Model Recursive Estimation of Images," Proc. of 1979 ICASSP, pp. 642-645, April, 1979.

14. C. II. Chen, "Adaptive Image Filtering," Proc. of IEEE Pattern Recognition and Image Processing Conference, pp. 32-37, August, 1979.

15. R. M. Haralick, K. Shanmugam, and I. Dinstein, "Textural Features for Image Classification," IEEE Trans. on Systems, Man and Cybernetics, SMC-13, pp. 610-621, 1973.

16. M. S. Bartlett, "The Statistical Analysis of Spatial Patterns," Chapman and Hall, 1975.

17. R. Chellappa, R. L. Kashyap and N. Ahuja, "Decision Rules for Choice of Neighbors in Random Field Models of Images," TR-802, Computer Science Center, University of Maryland, August, 1979.

18. J. T. Tou, D. B. Kao and Y. S. Chang, "Pictorial Texture Analysis and Synthesis," Proc. of the 3rd International Joint Conference on Pattern Recognition, 1976.

19. B. R. Hunt, "Bayesian Methods in Non-Linear Digital Image Restoration," IEEE Trans. on Computers, Vol. C-26, pp. 219-229, March, 1977.

20. B. R. Hunt and T. M. Cannon, "Non-Stationary Assumptions for Gaussian Models of Images," IEEE Trans. on Systems, Man and Cybernetics, SMC-6, pp. 876-882, December, 1976.

21. G. Matheron, "The Theory of Regionalized Variables and Its Applications," Les Cahiers du Centre de Morphologie Math. de Fontainbleu, Vol. 5, 1971.

22. K. Deguchi and I. Morishita, "Texture Characterization and Texture-Based Image Partitioning Using Tw-Dimensional Linear Estimation Techniques," IEEE Trans. on Computers, Vol. C-27, August, 1978.

23. A Rosenfeld and A. C. Kak, "Digital Picture Processing," Academic Press, Inc., 1976.

ADDITIONAL REFERENCES

1. N. Ahuja and A. Rosenfeld, "Image Models," TR-731, Computer Vision Laboratory, University of Maryland, July, 1979.

2. N. Ahuja, "Mosaic Models for Image Analysis and Synthesis," Ph.D. Dissertation, Department of Computer Science, University of Maryland, 1979.

3. D. R. Brillinger, "Fourier Analysis of Stationary Processes," Proc. of IEEE, Vol. 62, pp. 1628-1642, 1974.

4. R. Chellappa and N. Ahuja, "Statistical Inference Theory Applied to Image Modeling," University of Maryland Computer Science Center, TR-745, March, 1979.

5. K. Deguchi and I. Morishita, "Image Coding and Reconstruction by Two-Dimensional Optimal Linear Estimation," Proc. of the 4th IJCPR, pp. 530-532, November, 1978.

6. A. Gagalowicz, "Analysis of Texture Using a Stochastic Model," Proc. of the 4th IJCPR, pp. 541-544, November, 1978.

7. A. Habibi, "Two-Dimensional Bayesian Estimate of Images," Proc. of IEEE, Vol. 60, pp. 878-883, 1972.

8. A. K. Jain and E. Angel, "Image Restoration, Modelling, and Reduction of Dimensionality," IEEE Trans. on Computers, Vol. C-23, pp. 470-476, May, 1974.

9. A. K. Jain, "A Semi-Causal Model for Recursive Filtering of Two-Dimensional Images," IEEE Trans. on Computers, Vol. C-26, pp. 343-350, 1977.

10. M. Kanefsky and M. G. Strintzis, "A Decision Theory Approach to Picture Processing," IEEE Trans. on Computers, Vol. C-27, No. 1, pp. 32-36, January, 1978.

11. J. S. Lee, "Digital Image Processing by Use of Local Statistics," Proc. of IEEE Conference on Pattern Recognition and Image Processing, May, 1978.

12. D. H. McCormick and S. N. Jayaramamurthy, "Time Series Model for Texture Synthesis," International Journal of Computer and Information Sciences, Vol. 3, pp. 329-343, 1974.

13. H. E. Nahi and S. Lopez-Mora, "Estimation and Detection of Object Boundaries in Noisy Images," IEEE Trans. on Automatic Control, AC-23, pp. 834-845, October, 1978.

14. D. P. Panda and A. C. Kak, "Recursive Least Squares Smoothing of Noise In Images," IEEE Trans. on Acoustics, Speech and Signal Processing, ASSP-25, pp. 520-524, 1977.

15. W. K. Pratt, O. D. Faugeras, and A. Gaglowicz, "Visual Discrimination of Stochastic Texture Fields," IEEE Trans. on Systems, Man and Cybernetics, SMC-8, pp. 796-804, 1978.

16. A. Rosenfeld and L. S. Davis, "Image Segmentation and Image Models," Proc. of the IEEE, Vol. 67, No. 5, pp. 764-772, May, 1979.

17. W. B. Thompson, "Texture Boundary Analysis," IEEE Trans. on Computers, Vol. C-26, pp. 272-276, 1977.

18. P. Whittle, "On Stationary Process in the Plane," Biometrika, Vol. 41, pp. 434-449, 1954.

19. H. Akaike, "Time Series Analysis and Control Through Parametric Models," in "Applied Time Series Analysis" edited by D. F. Findley, Academic Press, Inc., 1978.

20. W. E. Larimore, "Statistical Inference on Stationary Random Field," Proc. of IEEE, Vol. 65, pp. 961-970, 1977.

22. R. L. Kashyap, "Two-Dimensional Autoregressive Models for Images: Parameter Estimation and Choice of Neighbors," Proc. Workshop on Pattern Recognition and Artificial Intelligence, April, 1978.

22. H. W. Penn and M. Kanefsky, "Optimum Linear Smoothing of Noisy Digitized Chromosome Images," Proc. Workshop on Pattern Recognition and Artificial Intelligence, April, 1978.